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# ON A QUALITATIVE DIFFERENCE BETWEEN THE DYNAMICS OF PARTICLE PRODUCTION IN SOFT AND HARD PROCESSES OF HIGH ENERGY COLLISIONS

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## ABSTRACT

The qualitative difference between the anomalous scaling properties of hadronic final states in soft and hard processes of high energy collisions is studied in some detail. It is pointed out that the experimental data of $e^+e^-$collisions at $E_{\rm cm}$ =91.2 GeV from DELPHI indicate that the dynamical fluctuations in $e^+e^-$collisions are isotropical, in contrast to the anisotropical fluctuations oberserved in hadron-hadron collision experiments. This assertion is confirmed by the Monte Carlo simulation using the Jetset7.4 event generator.

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Quantum Chromodynamics (QCD) [1] as a candidate of the basic theory of strong interaction has been very successful in the study of hard processes, such as scaling violation in deep inelastic scattering (DIS), hadronic-jet production in $e^+e^-$annihilation, large-$p_t$-jet production in hadron-hadron collisions etc.. In such processes there exist large scales so that QCD can be solved perturbatively due to asymptotic freedom.

On the contrary, in soft-hadronic processes, e.g. hadron-hadron, hadron-nucleus and nucleus-nucleus collisions below CERN collider energies, there is no large scale present, and the interaction becomes so strong that pQCD is no longer applicable. In these cases, the problem is hard to be solved analytically from the first principle and phenomenological regularities extracted from experiments have to be ultilized to increase our knowledge on the property of the basic dynamics in such processes.

In this respect, a comparison of the phenomenology of multiparticle final states in soft and hard processes is worthwhile. Such comparison is helpful in getting information about the similarity and distinction between the dynamics of particle production in these two kinds of processes. In particular, to find out the qualitative differences, if any, between them is especially interesting.

In this paper we will show that such a qualitative difference does exist in the anomalous scaling property of final-state particle distribution. It turns out that the higher-dimensional factorial moments (FM) do have anomalous scaling property (obey a power law with the diminishing of phase space) when and only when the partition of phase space is anisotropic for soft processes while isotropic for hard ones. This means that the dynamical fluctuations are anisotropic in the former case while isotropic in the latter. This qualitative distinction may serve as a useful criterion in the study of the basic dynamics of particle production in these two kinds of processes.

Let us first review briefly the status of dynamical-fluctuation study in high energy collisions [2]. The interest in this study was first stimulated by the experimental finding in 1983 of unexpectedly large local fluctuations (2 times the average) in a high multiplicity event from JACEE [3]. The same phenomena were observed latter also in accelerator experiments, the local fluctuations being as large as 60 times the average[4]. Such large fluctuations may not be simply due to statistical reason and is an indication of the existence of non-statistical (dynamical) fluctuations.

In order to eliminate the statistical fluctuations and study the dynamical ones, Białas and Peschanski [5] proposed to make use of the normalized factorial moments (FM)

$$F_q(M) = \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}, \tag{1}$$

where a region $\Delta$ in 1-, 2- or 3-dimensional phase space is divided into $M$ cells, $n_m$ is the multiplicity in the $m$th cell, and $\langle\cdots\rangle$ denotes vertically averaging over the event sample. They have been able to prove that if dynamical fluctuations do exist, the factorial moments will show power-law (anomalous scaling) behavior

$$F_q(M) \propto (M)^{\phi_q} \qquad (M\to\infty)\ , \tag{2}$$

which can be represented as a straight line in the $\ln F_q$ vs. $\ln M$ plot. Such kind of power-law scaling is typical for fractals [6].

As known in the fractal theory [7], when the underlying space is of higher (2 or 3) dimension, there are qualitatively two different kinds of fractals — self-similar and self-affine. The difference between these two kinds of fractals lies in the scales used in different space directions. If one and the same scale is used in all the directions, i.e. if the power law (2) holds when the space is diminished similarly in different directions, the fractal is called *self-similar*. On the contrary, if different scales are used in different directions, i.e. if the power law (2) holds when and only when the space is diminished with different ratios in different directions, the corresponding fractal is called *self-affine*. An example of the latter is the three dimensional space of landscape, where the vertical direction is a special one due to the existence of gravity, which causes the vertical variations of landscapes to be scaled differently from the horizontal ones.

It turns out that the dynamical fluctuations in the soft-hadronic interactions are similar to the 3-D space of landscape in the sense that the longitudinal direction (the direction of the momenta of incident hadrons) is privileged so that the higher-dimensional FM scales when and only when the phase space is partitioned anisotropically with different partition numbers in longitudinal and transverse directions [8]. In Fig's.1($a$) and ($b$) are shown the results from NA22 [9] and NA27 [10] respectively.

In order to characterize the different ways of phase space partition, a parameter $H$ called Hurst exponent has been used, which is defined as

$$H_{ab} = \frac{\ln M_a}{\ln M_b}, \tag{3}$$

where $M_a$ and $M_b$ are the partition numbers in directions $a$ and $b$ respectively. $H = 1$ means that the phase space is divided similarly in directions $a$ and $b$, while $H \neq 1$ characterizes the anisotropical way of phase space partition, cf. Fig.2.

It can be seen from Fig's.1($a$) and ($b$) that the log-log plots of the second order FM ($F_2$) versus the partition number ($M$) in hadron-hadron collisions, instead of being straight, are bending upwards when the phase space is divided similarly in different directions ($H = 1$). On the other hand, if the phase space is divided anisotropically with some particular values of $H$ — $H_{\parallel\perp} = 0.475, H_{p_t\varphi} = 1$ for NA22, $H_{\eta\varphi} = 0.74$ for NA27 — the log-log plots become straight within the experimental error. These results show that the dynamical fluctuations in soft-hadronic interactions are anisotropical and the corresponding fractal is self-affine.

However, the published results of hard collisions, e.g. the results of $e^+e^-$annihilation at $E_{\rm cm} = 91.2$GeV from DELPHI [11], showed a sharply different situation. In this case, the 3-D $\ln F_2$ vs. $\ln M$ plot under isotropical phase space partition ($H = 1$) follows a straight line reasonably well, cf. Fig.1($c$), provided the first point is omitted to reduce the influence of momentum conservation [12]. This remarkable fact indicates that dynamical fluctuations in soft and hard processes are qualitatively different, being anisotropic in the former case while isotropic in the latter.

In order to confirm this observation, we have done a Monte Carlo simulation

with Lund JETSET7.4 event generator. In total 1,000,000 events are produced for $e^+e^-$collisions at $E_{\rm cm} = 91.2$GeV. The FM analysis is done both in the laboratory system and in the thrust system. In order to reduce the trivial effect of the non-flat of average distributions the cummulant variables have been used [13].

In Fig.3 and Table I are shown the results for lab. system. In this system the $z$ axis is chosen to be along the direction of motion of the incident particles ($e^+$ and $e^-$). Since these two particles have been annihilated into a single virtual particle ($Z^0$ or $\gamma^*$), their direction of motion is not privileged in the final state hadronic system any more. Therefore, it is meaningless to distinguish "longitudinal" and "transverse" directions in this coordinate system and so the Cartesian momenta $p_x, p_y, p_z$, instead of the "longitudinal Lorentz invariant" variables $y, p_{\rm t}, \varphi$, are used.

**Table I** The fitting parameters of 1-D FM in the lab. system

| Varibles | $A$ | $B$ | $\gamma$ | Omitting point(s) |
|---|---|---|---|---|
| $p_x$ | 1.217 ± 0.001 | 0.400 ± 0.004 | 1.278 ± 0.010 | 1 |
| $p_y$ | 1.218 ± 0.001 | 0.399 ± 0.004 | 1.267 ± 0.009 | 1 |
| $p_z$ | 1.214 ± 0.001 | 0.409 ± 0.004 | 1.368 ± 0.010 | 1 |

It can be seen from Fig.3$a$ that the log-log plot of 3-D $F_2$ versus $M$ fits a straight line well. The value of Hurst exponents can be obtained through fitting the three 1-D plots, cf. Fig.3$b$, to the formulae [14]

$$F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a}, \quad a = p_x, p_y, p_z \tag{4}$$

as

$$H_{ab} \equiv \frac{\ln \lambda_a}{\ln \lambda_b} = \frac{1+\gamma_b}{1+\gamma_a}. \tag{5}$$

The values of fitting parameters are listed in Table I and the resulting Hurst exponents:

$$H_{p_xp_y} = \frac{1+\gamma_{p_y}}{1+\gamma_{p_x}} = 0.995 \pm 0.008,$$

$$H_{p_zp_y} = \frac{1+\gamma_{p_y}}{1+\gamma_{p_z}} = 0.957 \pm 0.008,$$

$$H_{p_zp_x} = \frac{1+\gamma_{p_x}}{1+\gamma_{p_z}} = 0.962 \pm 0.008$$

are nearly equal to unity. All of these show clearly that the dynamical fluctuations are isotropical and the corresponding fractal is self-similar rather than self-affine.

As is well known, in high energy $e^+e^-$collisions hadronic jets are produced. The reason is that the virtual photon or $Z^0$ formed in $e^+e^-$annihilation first produce a pair

of fast moving quark-antiquark, and then the latter radiating gluons turned finally into hadrons. The direction of motion of the quark-antiquark pair is privileged in the final state hadronic system and can be chosen as the "longitudinal" direction. The longitudinal rapidity

$$y = \ln \frac{1}{2} \frac{E + p_{\|}}{E - p_{\|}} \tag{6}$$

can thus be defined, where $p_{\|}$ is the momentum along the "longitudinal" direction.

Experimentally, the direction of the quark-antiquark pair can be extracted from the momenta of the final-state particles as the direction determined by [15]

$$T = \max \frac{\sum_i |p_{\|i}|}{\sum_i |\vec{p}_i|}. \tag{7}$$

This direction is called *thrust axis*, and can be taken as the "longitudinal" direction. Using the same formula in the plan perpandicular to $T$, the *major axis* $T_2$ [16] is obtained. The direction perpendicular to both $T$ and $T_2$ is called the *minor axis* $T_3$.

The *thrust coordinate system* is defined as a Cartesian coordinate system, using the three axes $T_3$, $T_2$ and $T$ as $x$, $y$ and $z$ axes respectively. The longitudinally Lorentz invariant variables $y$, $p_{\mathrm{t}}$ and $\varphi$ are then obtained with the rapidity $y$ defined according to Eq.(5). The azimuthal angle $\varphi$ is defined relative to the $x$ ($T_3$) axis.

**Table II** The fitting parameters of 1-D FM in the thrust system

| Varibles | $A$ | $B$ | $\gamma$ | Omitting point(s) |
|---|---|---|---|---|
| $y$ | 1.468 ± 0.001 | 0.751 ± 0.003 | 0.887 ± 0.004 | 1 |
| $p_{\mathrm{t}}$ | 1.157 ± 0.001 | 0.130 ± 0.002 | 0.825 ± 0.017 | 1 |
| $\varphi$ | 1.033 ± 0.001 | 1.077 ± 0.037 | 2.742 ± 0.027 | 3 |

The resulting log-log plots are shown in Fig.4 and the fitting parameters are listed in Table II. It turns out that, inspite of the 3-D log-log plots being still straight, the three 1-D plot are very different, so that the three sets of fitting parameters differ sharply. In particular, the value of $\gamma_\varphi$ is more than 3 times bigger than $\gamma_y$ and $\gamma_{p_{\mathrm{t}}}$, which would mean $H_{y\varphi}, H_{p_{\mathrm{t}}\varphi} \gg 1$, cf. Eq.(5). However, this is not really the case.

Let us notice that there are 3 parameters in the projection formula Eq.(4). In getting the value of $H$, only the fitting parameters $\gamma$ are used, cf. Eq.(5), but this does not mean that the values of the other two parameters $A$ and $B$ are unimportant. The projection formulae for these two parameters are [17]:

$$A = \frac{\lambda - 1}{\lambda - C^{(2)}}, \qquad B = \frac{C^{(2)} - 1}{\lambda - C^{(2)}}, \tag{8}$$

where $C^{(2)} = \langle \omega^2 \rangle / \langle \omega \rangle^2$ is the normalised second order moment of the probability $\omega$ of elementary space partition; $\lambda$ is the elementary partition number. It is evident that the parameters $A$ and $B$ satisfy a relation:

$$A - B = 1. \tag{9}$$

This relation should be approximately valid for any physically meaningful set of parameters. In Fig.5 are shown the difference $A-B$ for all the presently available cases. It can be seen that, in all the cases except for variable $\varphi$ in the thrust system of Jetset, relation (9) holds approximately, while for the latter case, this relation is sharply violated. This means that the set of fitting values listed in Table II is physically meaningless, and therefore it is impossible to draw any conclusion about the value of Hurst exponent from it.

The reason for the exceptional nature of the dynamical fluctuations for variable $\varphi$ in the thrust system is understandable. It is because the relative point for counting the value of $\varphi$, i.e. the $x$ axis, in the thrust system is fixed at the third thrust axis $T_3$. As a crude estimation, the second thrust axis $T_2$ is approximately the direction of the first hard gluon emitted by the quark or antiquark. To count $\varphi$ from $T_3$ means that the azimuthal angle of first hard gluon emission is fixed to 90 degree. Thus the fluctuations of the direction of first hard gluon emission are largely reduced.

In order to study the real 3-D fluctuations, it is necessary to loosen the correlation between the direction of first hard gluon emission and the $x$ axis. For this purpose, putting the $z$ axis still on the main thrust axis, we turn the coordinate system arround it and let the new $x$ axis lies on the $z_0$-$z$ plan, where $x_0, y_0, z_0$ denote the axes of the lab system and $x, y, z$ those of the turned system, as shown in Fig.6.

The log-log plots in the turned system are shown in Fig.7 and the fitting parameters are listed in Table III. The 3-D plot is still straight. The three 1-D plots become similar and the differences between the fitting parameters $A$ and $B$ for these three plots, especially that for the variable $\varphi$, become near to unity, cf. Fig.5. The Hurst exponents calculated from the fitting parameters:

$$H_{yp_{\rm t}} = \frac{1+\gamma_{p_{\rm t}}}{1+\gamma_y} = 0.95 \pm 0.02,$$

$$H_{yp_\varphi} = \frac{1+\gamma_\varphi}{1+\gamma_y} = 1.11 \pm 0.02,$$

$$H_{p_{\rm t}\varphi} = \frac{1+\gamma_\varphi}{1+\gamma_{p_{\rm t}}} = 1.18 \pm 0.03$$

are approximately equal to unity. This confirms further that the dynamical fluctuations are isotropical in $e^+e^-$collisions and the corresponding fractal is self-similar.

**Table III** The fitting parameters of 1-D FM in the turned system

| Varibles | $A$ | $B$ | $\gamma$ | Omitting point(s) |
|---|---|---|---|---|
| $y$ | 1.469 ± 0.001 | 0.755 ± 0.002 | 0.890 ± 0.004 | 1 |
| $p_{\rm t}$ | 1.160 ± 0.001 | 0.131 ± 0.002 | 0.785 ± 0.014 | 1 |
| $\varphi$ | 1.175 ± 0.001 | 0.492 ± 0.008 | 1.102 ± 0.010 | 3 |

In conclusion, in this paper a qualitative difference between the dynamics of particle production in soft and hard processes of high energy collisions is analysed in some detail. It is pointed out that the experimental data of $e^+e^-$collisions at $E_{\rm cm}$ =91.2 GeV from DELPHI indicate that the dynamical fluctuations in the hadronic final states of this experiment are isotropical, in contrast to the ansotropical fluictuations oberserved in hadron-hadron collision experiments (NA22 and NA27). This observation is confirmed using the Jetset7.4 event generator. The 3-D factorial moments (FM) in the Monte Carlo sample with 1000000 events at $E_{\rm cm}$ =91.2 GeV shows good scaling when the phase space is divided isotropically. The three 1-D FM's in the lab frame have similar shapes. The corresponding Hurst exponents are nearly equal to unity. In the thrust system, after turning the transverse axes to eliminate the correlation with the direction of first hard gluon emission, the 1-D FM's become similar too, and the Hurst exponents are approximately equal to unity.

Let us discuss briefly why the multiparticle final states in $e^+e^-$collisions are isotropic (self-similar) instead of anisotropic (self-affine) fractal.

It has long been recognized [18] that QCD branching has fractal structure. Being a hard process with large $Q^2$, the "branches" are developed isotropically in 3-D phase space without a limited $p_{\rm t}$ [18]. The resulting parton distribution is naturally a self-similar (isotropic) fractal. What are observed in experiments are, of course, not the partons themselves, but the final state hadrons produced from them. However, the hadronization of each parton is a soft process with limited $p_{\rm t}$ perpendicular to the jet axis. These axes are distributed in 3-D phase space as an isotropic fractal, without privileged direction. Therefore, the anisotropic effects of the hadronization of different partons cancell each other, and the isotropic (self-similar) fractal of parton-distribution will be largely survived after hadronization. This is why an approximately self-similar fractal can be observed in $e^+e^-$collisions.

The qualitatively different fractal properties between the hadronic final states of hard and soft collisions (self-similar and self-affine) may serve as a useful criterion in the study of the basic dynamics of particle production in these two kinds of processes.

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# Figure Captions

**Fig.1** Qualitative difference in the scaling property of factorial moments in h-h and $e^+e^-$ collisions.

**Fig.2** A sketch of the different ways of phase space division.
$(a)$ isotropic; $(b)$ anisotropic, $H < 1$; $(c)$ anisotropic, $H > 1$.

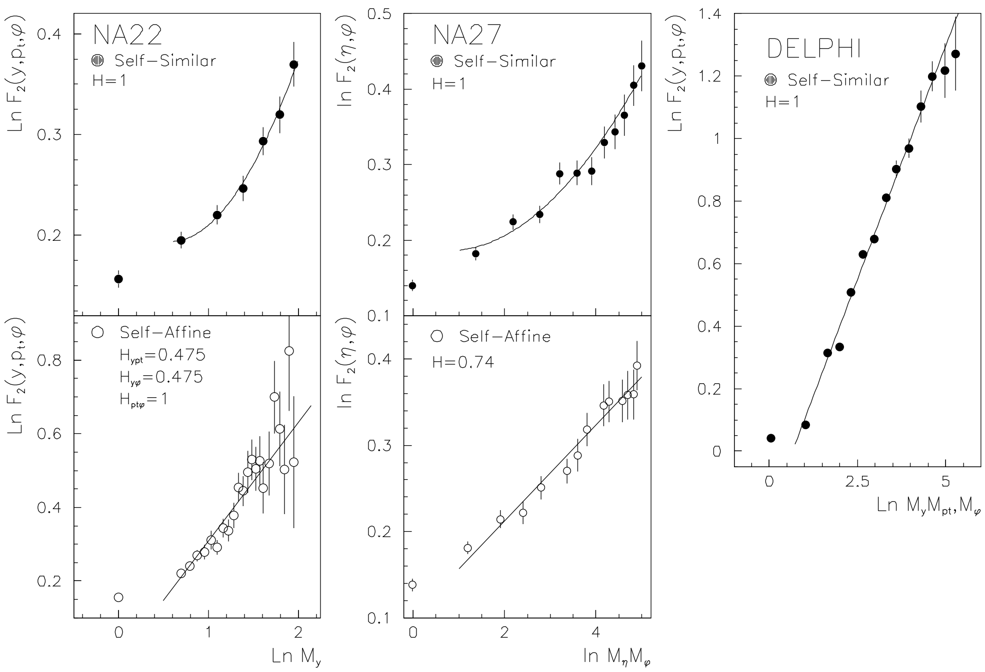
**Fig.3** Factorial moments of Jetset Monte Carlo sample in the lab frame.

**Fig.4** Factorial moments of Jetset Monte Carlo sample in the thrust frame

**Fig.5** Compilation of the values of $A - B$.

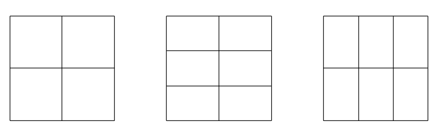
**Fig.6** The turned coordinate system.

**Fig.7** Factorial moments of Jetset Monte Carlo sample in the turned frame.



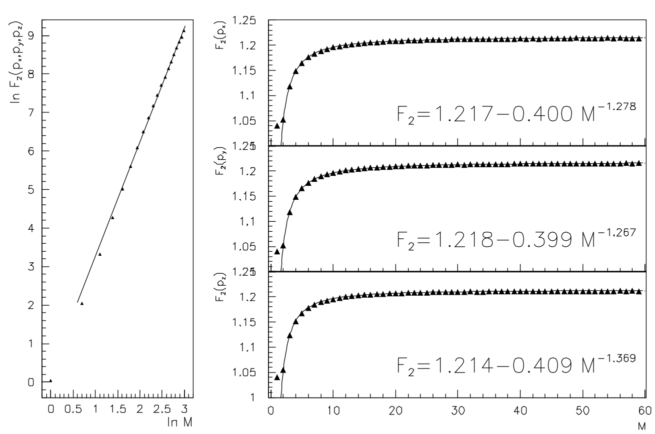

$(a)$ $(b)$ $(c)$

**Fig.1**



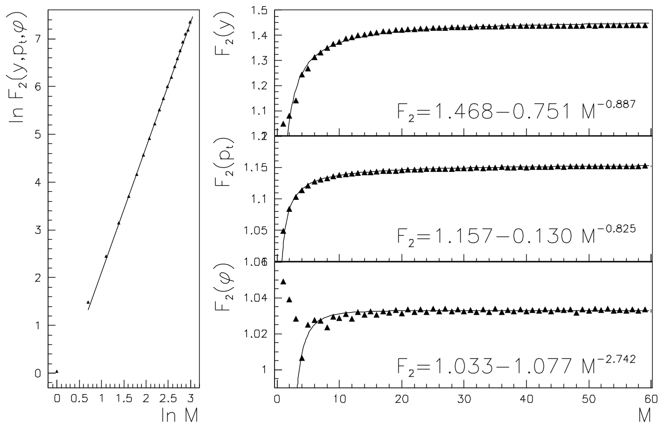
$(a)$ $(b)$ $(c)$

**Fig.2**



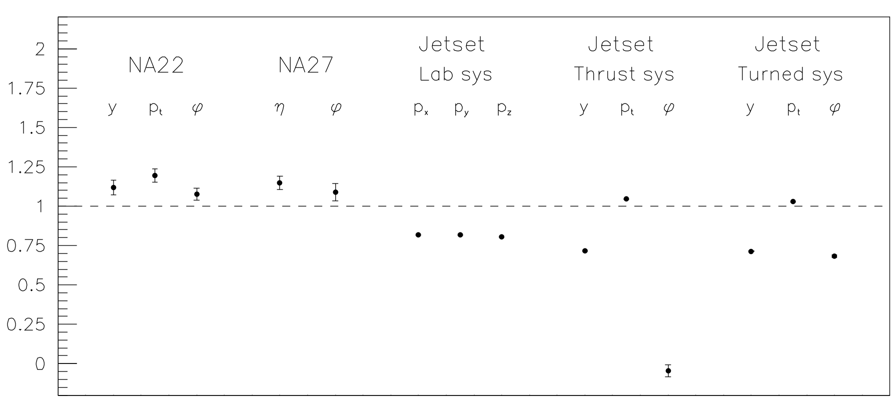

$(a)$ $(b)$

**Fig.3**



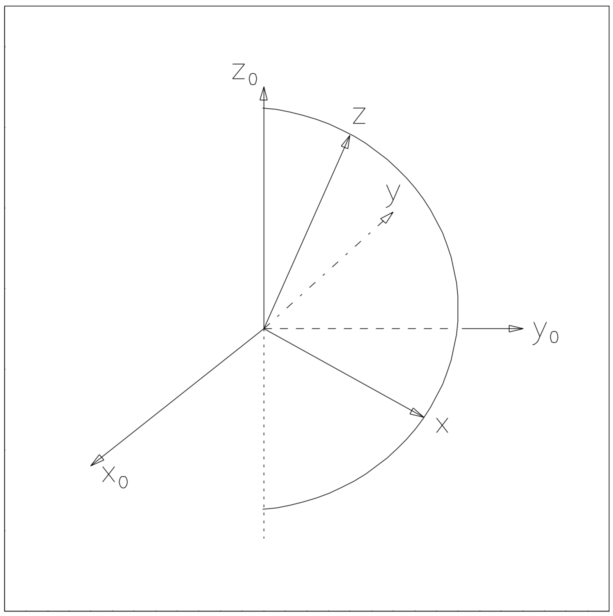
ln F2(y,pt,φ)
ln M
F2(y)
F2=1.468−0.751 M^−0.887
F2(pt)
F2=1.157−0.130 M^−0.825
F2(φ)
F2=1.033−1.077 M^−2.742
M


**Fig.4**



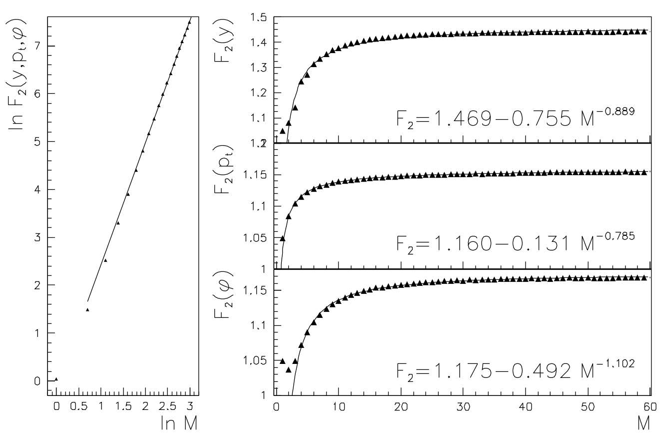
NA22
NA27
Jetset
Lab sys
Jetset
Thrust sys
Jetset
Turned sys
y pt φ
η φ
px py pz
y pt φ
y pt φ


**Fig.5**



$Z_0$
$Z$
$y$
$y_0$
$x$
$x_0$


**Fig.6**



ln $F_2(y,p_t,\varphi)$
ln M
$F_2(y)$
$F_2$=1.469−0.755 $M^{-0.889}$
$F_2(p_t)$
$F_2$=1.160−0.131 $M^{-0.785}$
$F_2(\varphi)$
$F_2$=1.175−0.492 $M^{-1.102}$
M


**Fig.7**